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CONTACT HEAT CONDUCTION THROUGH PERIODICALLY CONTACTING RODS

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The process of heat conduction through periodically contacting rods has been considered. On the basis of the method of eigenfunction expansion of the boundary-value problem an evolutionary algorithm for constructing a solution for any cycle of contacting has been proposed, an explicit solution for the first cycle has been found, and a computer program of temperature field calculation with a given accuracy from the viewpoint of the proposed mathematical model has been developed. The dependence of the total heat resistance to the thermal flow through periodically contacting metallic surfaces of two rods in the presence or absence heat resistance on the material of the contact pair, the temperature conditions, the landed forces, the geometry of the contacting surfaces, and the contact frequency and time has been established experimentally.

Introduction. Contact heat transfer has been the subject of a large number of domestic and foreign works [1–3]. Mainly the processes of heat transfer in technical systems with static contacts were investigated. At the same time in steam- and gas-turbine plants, in internal combustion engines, in hot plastic metal working, and in soldering operations, contact pairs with periodically contacting metallic surfaces through which high-density thermal flows pass are used. The process of designing, manufacturing, and using such systems require information on the formation of temperature fields, characteristic features of the heat exchange, and measures to control the heat transfer processes.

The investigation of this rather nontrivial, in both the theoretical and experimental aspects, problem was initiated in [4–6]. The results of these works point to a variety of factors influencing the process of heat transfer through such joints. This is evidenced by the fact that the authors give preference to the analog computer for investigating one-dimensional heat transfer between two periodically contacting rods.

Formulation of the Problem and Its Solution. We have two metal rods of equal length, of which the right one is located on the segment $[0, l]$ and the left one on $[-l, 0]$. The distant ends of the rods always stay heated to temperatures T_0 and $(-T_0)$. When the rods come in contact with each other, the end temperatures instantaneously become equal and stay zero all the the time at t — $[0, t_1)$, where t_1 is the contact duration. For the sake of simplicity of the analysis, let us refer the temperatures of both rods to T_0, and the coordinate along the rods $\tilde{x}$ to the rod length l, i.e.,

$$U(\tilde{x}, t) = T_0 u(x, t) , \quad \tilde{x} = lx , \tag{1}$$

where $u(x, t)$ and x are the dimensionless temperature and coordinate. The time t remains a dimensional quantity. Assume that for the right rod at $0 \le x \le 1$ the temperature field is described by some function $u(x, t)$, and in the left rod at $-1 \le x \le 0$ it is asymmetric and, therefore, will be defined by the function $[-u(x, t)]$. Such an assumption simplifies the whole problem, since it is enough to formulate it only for the right rod.

Let us denote by $u_1(x, t)$ the temperature in the right rod at the first contact of the rods, by $u_2(x, t)$ the temperature at the second contact after their separation, by $u_i(x, t)$ the temperature during the ith contact after the $(i-1)$th separation and by $\mu_i(x, t)$ the temperature in the right rod in the separated state after the ith contact of the rods. Denote by t_1 the time when the rods are in contact and by t_2 the time of their contactless state. In considering the cy-

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cles, we will each time count off the time t from zero. In the contacting state of the rods, $0 \le t \le t_1$, and in their separated state $0 \le t \le t_2$ for all i numbers.

Let the rods contact each other during the time t_1 in the ith cycle. Then we arrive at the following initial-boundary-value problem: solve the heat conduction equation

$$\frac{\partial u_i}{\partial t} = \frac{a^2}{l^2} \frac{\partial^2 u_i}{\partial x^2} , \quad t — [0, t_1] , \quad x — [0, 1] , \quad i = 1, 2, ... , \tag{2}$$

satisfying the given initial condition

$$u_i \big|_{t=0} = f_i (x) , \quad f_1 (x) = 1 , \quad f_i (1) = 1 , \quad i = 1, 2, ... , \tag{3}$$

and the boundary conditions

$$u_i \big|_{x=0} = 0 , \quad u_i \big|_{x=1} = 1 , \quad i = 1, 2, ... \tag{4}$$

If contact between the rods is absent, then their close ends do not exchange heat with each other. For the out-of-contact state of the two rods, we will have another initial-boundary-value problem: solve the heat conduction equation

$$\frac{\partial \mu_i}{\partial t} = \frac{a^2}{l^2} \frac{\partial^2 \mu_i}{\partial x^2} , \quad t — [0, t_2] , \quad x — [0, 1] , \tag{5}$$

satisfying some initial condition

$$\mu_i \big|_{t=0} = \varphi_i (x) , \quad \varphi_i (1) = 1 , \quad i = 1, 2, ... , \tag{6}$$

and the boundary conditions

$$\left. \frac{\partial \mu_i}{\partial x} \right|_{x=0} = 0 , \quad \mu_i \big|_{x=1} = 1 , \quad i = 1, 2, ... \tag{7}$$

The process of contact heat conduction of two interacting rods begins with their first contact when the temperature field in them is described by the solution of the problem consisting of system (2)–(4) at $i = 1$. After time t_1 the rods move apart and during time t_2 the temperature field in them will be described by the solution of system (5)–(7) at $i = 1$. The final temperature field during the period of contact of the rods will act as the initial temperature for their subsequent out-of-contact state, i.e.,

$$u_i (x, t_1) = \mu_i \big|_{t=0} , \quad i = 1, 2, ...$$

Therefore, each function $\varphi_i(x)$ as the initial conditions in problems (5)–(7) should be calculated by the formula

$$\varphi_i (x) = u_i (x, t_1) , \quad i = 1, 2, ... \tag{8}$$

Likewise, the final temperature field of the rods in the contactless period will serve as the initial temperature for their subsequent contact state:

$$\mu_i (x, t_2) = u_{i+1} \big|_{t=0} , \quad i = 1, 2, ...$$

Thus, we will take the functions $f_i(x)$ used as the initial conditions for problems (2)–(4) to be equal to

$$f_1 (x) = 1 , \quad f_i (x) = \mu_{i-1} (x, t_2) , \quad i = 2, 3, ... \tag{9}$$

The temperature fields of the rods in their contact and out-of-contact states will influence each other through the initial conditions (8) and (9) in the following sequence: 1) solve problem (2)–(4) at $i = 1$ with the initial condition $u_1(x, t)\,|_{t=0} = 1$; 2) solve problem (5)–(7) at $i = 1$ where $\varphi_1(x) = u_1(x, t_1)$; 3) solve problem (2)–(4) at $i = 2$ with the initial condition from (9) at $i = 2$; 4) solve problem (5)–(7) at $i = 2$ with the initial condition from (8) at $i = 2$; 5) solve problem (2)–(4) at $i = 3$ with the initial condition from (9) at $i = 3$; 6) solve problem (5)–(7) at $i = 3$ with the initial condition from (8) at $i = 3$.

For contacs of rods with a given i number: 1) solve problem (2)–(4) with the initial condition from (9); 2) solve problem (5)–(7) with the initial condition from (8). We solve problem (2)–(4) when the rods are in contact with each other by the method of separation of variables. In this case, the expressions for $f_i(x)$ are taken from (9).

To bring the boundary conditions of the problem to a homogeneous form, we make the substitution

$$u_i = x + \tilde{u}_i\,, \quad i = 1, 2, \dots, \tag{10}$$

where $\tilde{u}_i(x, t)$ is a new unknown function. Substituting u_i from (10) into system (2)–(4), we obtain the following problem for $\tilde{u}_i$:

$$\frac{\partial \tilde{u}_i}{\partial t} = \frac{a^2}{l^2} \frac{\partial^2 \tilde{u}_i}{\partial x^2}\,, \quad t - [0, t_1]\,, \quad x - [0, 1]\,, \quad i = 1, 2, \dots; \tag{11}$$

$$\tilde{u}_i\,|_{t=0} = f_i(x) - x\,, \quad i = 1, 2, \dots; \tag{12}$$

$$\tilde{u}_i\,|_{x=0} = 0\,, \quad \tilde{u}_i\,|_{x=1} = 0\,, \quad i = 1, 2, \dots . \tag{13}$$

Instead of system (2)–(4) we have obtained a problem consisting of system (11)–(13) with homogeneous boundary conditions (13). Its solution by the method of separation of variables reduces to the expression of the rapidly converging series

$$\tilde{u}_i(x, t) = \sum_{n=1}^{\infty} A_{in} \exp\left[-\left(\frac{an\pi}{l}\right)^2 t\right] \sin n\pi x\,. \tag{14}$$

We find the constants A_{in} by means of the initial condition (12) upon expansion into a Fourier series in terms of the functions $\{\sin n\pi x\}$. However, we note first the inconsistency of the initial condition (12) and the boundary conditions (13) at $x = 0$, since in the general case

$$f_i(0) \neq 0\,. \tag{15}$$

The functions $[f_i(x) - x]$ from the initial condition (12) at $x = 0$ do not go to zero, as is required by the initial conditions (13). From the theory of Fourier series it is known that [7]: if some function $f^*(x)$ at the ends of the segment [0, 1] becomes zero

$$f^*(0) = 0 \quad \text{and} \quad f^*(1) = 0\,, \tag{16}$$

then its Fourier series in $\{\sin n\pi x\}$ rapidly converges throughout the segment [0, 1]. But if at least one inequality

$$f^*(0) \neq 0\,, \quad f^*(1) = 0\,, \tag{17}$$

takes place, then the Fourier series for such $f^*(x)$ at the point $x = 0$ diverges, and in the vicinity of the point $x = 0$ it converges slowly. For the other points the Fourier series for $f^*(x)$ will converge rapidly. The slow convergence of the series means that to obtain a reliable result in the vicinity of the point $x = 0$, one has to take into account, in the series used, a large number of terms, much larger than at the other points of the segment [0, 1]. This leads to additional difficulties in the numerical realization of the obtained solution of the problem. To lessen them partially, let us write an auxiliary expansion of the function $(1 - x)$ into a Fourier series in terms of the function $\{\sin n\pi x\}$ on the segment [0, 1]. This expansion can be found in the reference book [8]:

$$1 - x = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin n\pi x\,, \quad x — (0, 1]\,. \tag{18}$$

Series (18) slowly converges to unity in the vicinity of the point $x = 0$, and at the other points of the segment [0, 1] its convergence improves. Now we can proceed to the completion of constructing the solution $\tilde{u}_i$ of problem (11)–(13). Substituting $\tilde{u}_i$ from (14) into the initial condition (12), we obtain the equation

$$\sum_{n=1}^{\infty} A_{in}\sin n\pi x = f_i(x) - x\,. \tag{19}$$

Hence we get

$$A_{in} = 2\int_0^1 [f_i(x) - x]\sin n\pi x dx\,, \quad i = 1, 2, \ldots\,. \tag{20}$$

Since $f_1(x) = 1$, with the help of expansion (18) for A_{1n} we have the expression

$$A_{1n} = 2\int_0^1 (1 - x)\sin n\pi x dx = \frac{2}{n\pi} \tag{21}$$

and, therefore, from (10) we find a solution of problem (2)–(4) at $i = 1$ in explicit form

$$u_1(x, t) = 1 + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left[-\left(\frac{an\pi}{l}\right)^2 t\right]\sin n\pi x\,. \tag{22}$$

The algorithm of obtaining a solution of problem (2)–(4) at $i \geq 2$, when the rods are in contact with each other, has the following form: 1) the initial condition $f_i(x)$ should be taken from (9); 2) from (20) calculate A_{in}; 3) from (14) determine $\tilde{u}_i(x, t)$; 4) from (10) find $u_i(x, t)$.

Consider the solution of problem (5)–(7) when the rods are apart. In this case, boundary conditions of a mixed type hold, since the rod end at $x = 0$ is heat-insulated, and on the other end the temperature is held constant. We solve this problem, as the previous one, by the method of separation of variables. Therefore, it is necessary to bring the boundary conditions of problem (5)–(7) to a homogeneous form, which is achieved by the substitution

$$\mu_i(x, t) = 1 - \tilde{\mu}_i(x, t)\,, \tag{23}$$

where $\tilde{\mu}_i$ is a new unknown function. Substituting μ_i from (23) into system (5)–(7), we obtain for $\tilde{\mu}_i$:

$$\frac{\partial\tilde{\mu}_i}{\partial t} = \frac{a^2}{l^2}\frac{\partial^2\tilde{\mu}_i}{\partial x^2}\,, \quad t — [0, t_2]\,, \quad x — [0, 1]\,, \tag{24}$$

$$\tilde{\mu}_i\big|_{t=0} = 1 - \varphi_i(x)\,, \tag{25}$$

$$\left.\frac{\partial\tilde{\mu}_i}{\partial x}\right|_{x=0} = 0\,,\quad \tilde{\mu}_i\,|_{x=1} = 0\,. \tag{26}$$

Thus, instead of (5)–(7) we have a problem consisting of system (24)–(26) for another unknown $\tilde{\mu}_i$ with the homogeneous boundary conditions (26). By the method of separation of variables we obtain $\tilde{\mu}_i(x, t)$ in the form of the Fourier series

$$\tilde{\mu}_i\,(x, t) = \sum_{m=1}^{\infty} B_{im} \exp\left\{-\left[\frac{a\pi}{2l}\,(2m-1)\right]^2 t\right\} \cos\,(2m-1)\,\frac{\pi}{2}\,x\,. \tag{27}$$

We find the constants B_{im} by means of the initial condition (25). To this end, we write the auxiliary expansion of the function $(1 - x)$ into a Fourier series in terms of $\cos\,(2m-1)\frac{\pi}{2}x$ on the segment [0, 1]

$$1 - x = \frac{8}{\pi^2}\sum_{m=1}^{\infty}\frac{1}{(2m-1)^2}\cos\,(2m-1)\,\frac{\pi}{2}\,x\,,\quad x — [0, 1]\,. \tag{28}$$

To find B_{im}, substitute $\tilde{\mu}_i(x, t)$ from (27) into the initial condition (25)

$$\sum_{m=1}^{\infty} B_{im}\cos\,(2m-1)\,\frac{\pi}{2}\,x = 1 - \varphi_i\,(x)\,,\quad i = 1, 2, ...\,. \tag{29}$$

Since $\varphi_i(1) = 1$ for any i, the right-hand side of (25) agrees with the boundary condition (26) at $x = 1$, which provides a good convergence of the Fourier series (29) for the initial condition $(1 - \varphi_1(x))$ at this point. From (29) we get

$$B_{im} = 2\int_0^1 \left[1 - \varphi_i\,(x)\right]\cos\,(2m-1)\,\frac{\pi}{2}\,xdx\,. \tag{30}$$

Substituting B_{im} from (30) into (27), we obtain a solution for problem (5)–(7)

$$\mu_i\,(x, t) = 1 - \sum_{m=1}^{\infty} B_{im}\exp\left\{-\left[\frac{a\pi}{2l}\,(2m-1)\right]^2 t\right\}\cos\,(2m-1)\,\frac{\pi}{2}\,x\,,\quad i = 1, 2, ...\,. \tag{31}$$

The algorithm for solving problem (5)–(7) when the rods are apart is as follows: 1) find the function $\varphi_i(x)$ in the initial condition (6) from inequality (8); 2) determine B_{im} by formula (30); 3) find $\tilde{\mu}_i(x, t)$ from formula (27); 4) from formula (23) we get $\mu_i(x, t)$ — the solution of problem (5)–(7).

We now turn to the calculation of the temperature in the rods in the first and subsequent contact–separation cycles (i = 1, 2, ...). In realizing the above method of determining the temperature fields numerically, in the series we have to restrict oneselves to some finite number of terms. In the sums over the index n we take n = 1, 2, ..., N, and in the sums over the index m we assume m = 1, 2, ..., M. In expansion (18) used in solving problem (2)–(4) for contacting rods, the series on the right-hand side converges slowly in the vicinity of the point $x = 0$. Therefore, we define N as the number of terms in this series that has to be taken so that at $x = 10^{-1}$ the right- and the left-hand sides differ by no more than 10^{-3}. Hence we obtain the inequality for finding N:

$$\left|0.9 - \frac{2}{\pi}\sum_{n=1}^{N}\frac{1}{n}\sin\left(\frac{n\pi}{10}\right)\right| \le 10^{-3}\,. \tag{32}$$

In solving problem (5)–(7) when the rods are apart, expansion (28) is used. This series converges everywhere on the segment [0, 1]. At the point $x = 0$ the convergence of series (28) is the slowest; therefore, we determine the number of terms M in the sums with the index m under the condition that the error in calculating series (28) does not exceed 10^{-3}, i.e.,

$$\left| 1 - \frac{8}{\pi^2} \sum_{m=1}^{M} \frac{1}{(2m-1)^2} \right| \le 10^{-3} . \tag{33}$$

We obtain the expression for the temperature fields in the rods at their first contact after substituting A_{1n} from (21) into (12) at $i = 1$ and then into (10):

$$u_1(x, t) = x + \sum_{n=1}^{N} A_{1n} \exp\left[-\left(\frac{an\pi}{l} \right)^2 t \right] \sin n\pi x , \quad A_{1n} = \frac{2}{\pi n} . \tag{34}$$

Assuming $t = t_1$ in (34), from (8) we find the initial condition $\varphi_1(x)$ in explicit form for problem (5)–(7) when the rods are apart:

$$\varphi_1(x) = x + \sum_{n=1}^{N} A_{1n} \exp\left[-\left(\frac{an\pi}{l} \right)^2 t_1 \right] \sin n\pi x . \tag{35}$$

To find the solution $\mu_1(x, t)$, let us substitute $\varphi_1(x)$ from (35) into (36) at $i = 1$:

$$\sum_{m=1}^{M} B_{im} \cos (2m-1) \frac{\pi}{2} x = 1 - x - \sum_{n=1}^{N} A_{1n} \exp\left[-\left(\frac{an\pi}{l} \right)^2 t_1 \right] \sin n\pi x . \tag{36}$$

For further manipulations, the following integral will be needed:

$$\int_0^1 \sin n\pi x \cos (2m-1) \frac{\pi}{2} x dx = \frac{4}{\pi} \frac{n}{4n^2 - (2m-1)^2} . \tag{37}$$

Expanding the right-hand side of (36) into a Fourier series in $\left[\cos(2m-1)\frac{\pi}{2}x \right]$ on the segment [0, 1] by means of (28) and (37), we find B_{im}:

$$B_{im} = \frac{8}{\pi^2} \frac{1}{(2m-1)^2} - \frac{8}{\pi} \sum_{n=1}^{N} A_{in} \exp\left[-\left(\frac{an\pi}{l} \right)^2 t_1 \right] \frac{n}{4n^2 - (2m-1)^2} , \quad i = 1, 2, \dots . \tag{38}$$

The solution of problem (5)–(7), when the rods are separated, will have the form (31). In solving problem (2)–(4) for cycles $i \ge 2$ when the rods are in contact, substitute into the initial condition (20) the expression $f_i(x)$ from (2) and $\mu_{i-1}(x, t)$ from (21):

$$\sum_{n=1}^{\infty} A_{in} \sin n\pi x = 1 - x - \sum_{m=1}^{\infty} B_{(i-1)m} \exp\left\{ -\left[\frac{a\pi}{2l} (2m-1) \right]^2 t_2 \right\} \cos (2m-1) \frac{\pi}{2} x , \quad i = 2, 3, \dots . \tag{39}$$

Hence, with the use of (18) and integral (37) upon expanding the right-hand side of (39) in $\{\sin n\pi x\}$ on the segment (0, 1], we get

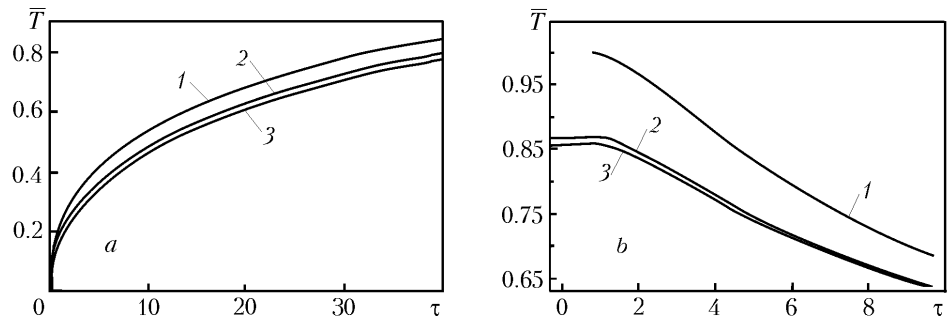


Fig. 1. Relative temperature distribution in the rod during three cycles (1–3) in the separated (a) and connected (b) states. τ, sec.

$$A_{in} = \frac{2}{\pi n} - \frac{8}{\pi} \sum_{m=1}^{M} B_{(i-1)m} \exp\left\{ - \left[\frac{a\pi}{2l} (2m-1) \right]^2 t_2 \right\} \frac{n}{4n^2 - (2m-1)^2}, \quad i = 2, 3, \ldots . \tag{40}$$

We find the solution of problem (2)–(4) for cycles $i \geq 2$, when the rods are in contact, from (10) and (14)

$$u_i = x + \sum_{n=1}^{\infty} A_{in} \exp\left[- \left(\frac{an\pi}{l} \right)^2 t \right] \sin n\pi x , \quad i = 2, 3, \ldots , \tag{41}$$

where A_{in} should be taken from (40).

The complete problem is solved by the following sequence of calculations: 1) find A_{1n} and $u_1(x, t)$ from (34) (1st cycle); 2) calculate B_{1m} by formula (38) and $\mu_1(x, t)$ from (31) at $i = 1$; 3) calculate A_{2n} form (40) and then $u_2(x, t)$ from (41) at $i = 2$; 4) calculate B_{2m} from (38) and then $\mu_2(x, t)$ from (31) at $i = 2$; 5) calculate A_{3n} from (40) and then $u_3(x, t)$ from (41) at $i = 3$; 6) calculate B_{3m} from (38) and then $\mu_3(x, t)$ from (31) at $i = 3$, and so on.

It should be noted that here the exact solution of the mathematical model of the investigated process of contact heat conduction is given. The discrepancies between the theoretical calculated results and experimental data may be due to all kinds of simplifications in the mathematical model, and they will accumulate with each cycles and can turn out to be significant if the number of cycles is large. Nevertheless, it is interesting to get an answer to the question after how many cycles the temperature profiles in the rod will begin to recur periodically and what is the form of these recurring profiles.

Figure 1 shows the numerical solutions of the analytical dependences obtained above. From the given values of the relative temperature in the rod it is seen that already in the third cycle the temperature profiles begin to recur periodically with to an accuracy of 10^{-3}.

Of no less practical interest is also the consideration of the process of formation of heat resistances in the contact zone of such periodically contacting rods.

Figure 2a shows the time-averaged temperature distributions in the "hot" rod in the places where temperature-sensitive elements are positioned. The given values of the temperatures along the length of the rod $T_{1\text{int}}$, ..., $T_{5\text{int}}$ have been obtained in the process of the experiment from the readings of the thermocouples for the regime of periodic contact under the condition of ideal conjugation of the rod surface when the contact heat resistance (CHR) can be neglected, i.e., $R_c \to 0$. There appears the possibility of graphical interpretation of the length of the segment l_{int} representing the heat resistance resulting from the periodic interruption of the thermal flow. In principle, across the thermal flow there arises a heat resistance created by the material of the rod of length l at a constant contact and a heat resistance due to the periodic interruption of the thermal flow, which can be imitated by a part of the rod of length l_{int}.

According to Fig. 2a, the losses of the heat flow due to its periodic interruption under the conditions of separation of the rods can be expressed analytically. In this case, the difference between the thermal flows for the stationary and quasi-stationary states will be written as

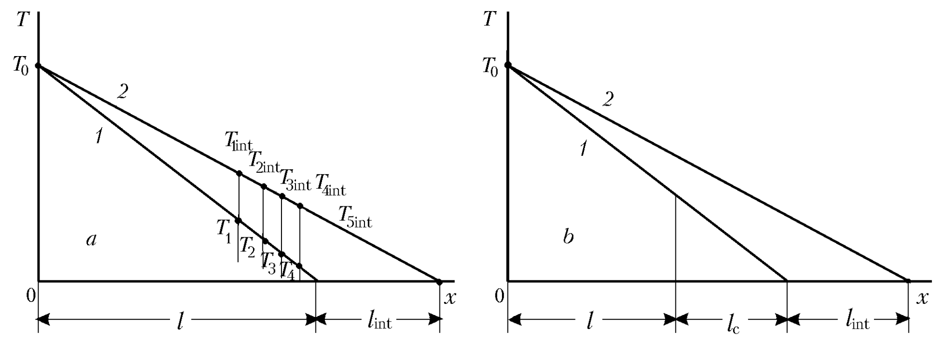


Fig. 2. Averaged temperature distribution in the "hot" rod in the absence (a) and presence (b) of CHR: 1) stationary state; 2) quasi-stationary state. T, K.

$$Q_{st} - Q_{q.st} = \frac{\lambda_m (T_{1int} - T_1)}{x_1 - x_0} = \frac{\lambda_m (T_{2int} - T_2)}{x_2 - x_0} = ... = \frac{\lambda_m (T_{5int} - T_5)}{x_5 - x_0},$$

hence in the dimensionless form

$$H = \frac{Q_{st} - Q_{q.st}}{Q_{st}} = \frac{T_{1int} - T_1}{T_0 - T_1} = ... = \frac{T_{5int} - T_5}{T_0 - T_5} = \frac{l_{int}}{l + l_{int}}, \tag{42}$$

where

$$l_{int} = \frac{lH}{1 - H}. \tag{43}$$

To describe the dependence of heat resistances, let us introduce dimensionless complexes $\nu l_{int}^2/a^2$ and νt_1 characterizing, respectively, the heat resistance due to the periodic interruption of the process of heat transfer when the rods are separated and the ratio of the contact length to the length of one cycle.

Of practical interest is the experimental establishment of the dependence

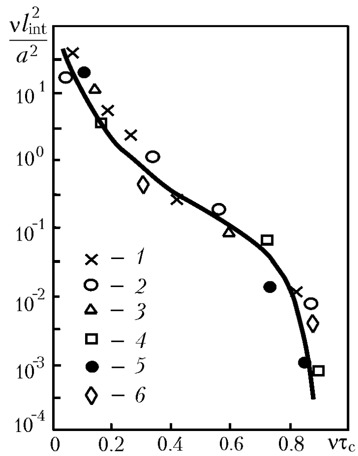
$$\frac{\nu l_{int}^2}{a^2} = f(\nu t_1). \tag{44}$$

The thermal model taking into account the influence of the CHR in the separation zone is closer to the real conditions. Figure 2b gives the temperature distribution in the "hot" rod in the stationary and intermittent states in the presence of a CHR. The appearance of one more variable in the form of a CHR characterized by l_c leads to the necessity of introducing a new dimensionless complex $\nu l_c^2/a^2$ in addition to the two complexes obtained earlier. Hence the dimensionless complex characterizing the heat resistance due to the periodic interruption of the thermal flow will be given in the form

$$\frac{\nu l_{int}^2}{a^2} = f\left[(\nu t_1), \left(\frac{\nu l_c^2}{a^2}\right)\right]. \tag{45}$$

Experimental Results and Discussion. Concrete forms of Eqs. (44) and (45) and the limits of their application can be established only by realizing a special experimental program. This is evident because the process of heat transfer through periodically contacting surfaces in the presence of CHR are influenced by the nature of the material of contact pairs, the temperature conditions, the landed forces, and the geometry of the surfaces.

The investigations were made on a setup consisting of a chamber in which two rods of diameter 24 mm and length 120 mm were placed. The rods are heat-insulated to decrease the radial heat loss. In the upper rod, an electric



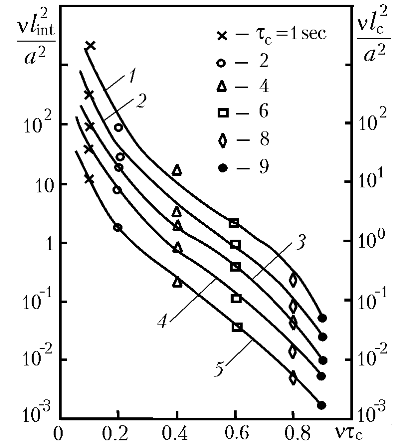


Fig. 3. Heat resistance due to the periodic interruption of the thermal flow versus the duration ratio between the contact and the period at different frequencies of contacts of the rods in the absence of CHR at the interface of the rods: 1) ν = 0.08; 2) 0.11; 3) 0.25; 4) 0.6; 5) 0.96; 6) 1.7 Hz.

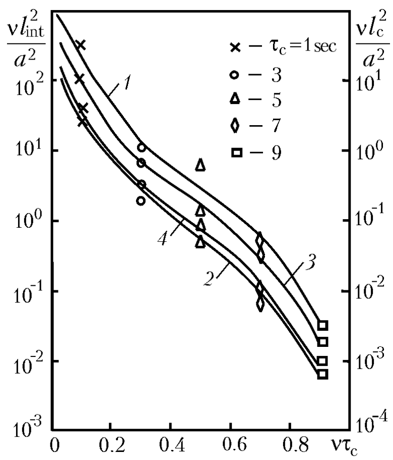
Fig. 4. Heat resistance due to the periodic interruption of the thermal flow versus the duration ratio between the contact and the period in the presence of CHR at the interface between the surfaces of rods from steel 12Kh18N10T at T_c = 413 K; P = 0.15 MPa; ν = 0.1 Hz: 1) the contact surfaces have been planed to R_z = 53 μm; 2) 32.4 μm; 3) milled; R_z = 18.3 μm; 4) 11 μm; 5) polished, R_z = 2.8 μm.

heater whose power is regulated by a rheostat is located. The constructional arrangement of the rod mounting allows for its replacement. With the help of the electric motor through the shaft with a cam the rod executes reciprocating motions. The contact time of the rods is controlled by a special timer unit which permits turning off the electric motor for a given period of time. The given pressure in the contact zone of the rods is held constant by expendable springs. The lower rod served as a cooler.

Mounted in each rod were four chromel-copel thermocouples. One more thermocouple was set in the immediate vicinity of the heater. The emf of the thermocouples was recorded by a potentiometer or an oscillograph. The experiments were performed in the following order. For each set of experiments, before switching on the heater, the electric motor was started and about 50 contact-separation cycles of the rods were carried out. After this mechanical training, the electric heater and the cooler were turned on. With the help of the rheostat and the timer the frequency and duration of contacts of the rods were given. Upon reaching pronounced quasi-stationary thermal conditions along the length of the rods the temperatures in the period of intimate contact and separation of the rod ends and the temperature T_0 in the region of the heater were measured (Fig. 2).

From the obtained temperature values, using expressions (42) and (43), we determined the length of the portion of the rod imitating the thermal resistance resulting from the periodic interruption of the thermal flow. The value of the rod length entering into (43) was taken from the concrete size of the hot rod to the thermocouple in the region of the heater.

Besides determining the heat resistance to the thermal flow for the moment of open contacts, we investigated the influence of the CHR at closed contacts on the formation of the total heat resistance. The known methods of experimental determination of the CHR [1, 3] are not acceptable in this case, since they are realized only under stationary thermal conditions. The method based on the modified variant of the technique proposed in [9] is used. Its specific feature is the establishment of regular cooling conditions, whose essence is described in [10]. The CHR value was determined on the same setup by the formula



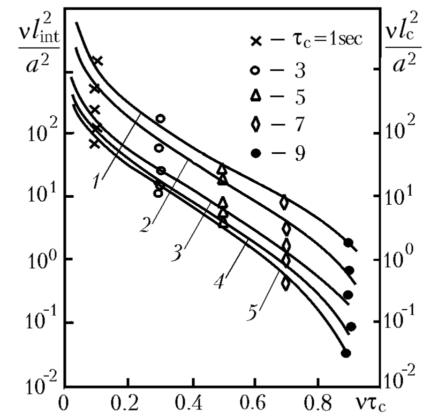


Fig. 5. Heat resistance due to the periodic interruption of the thermal flow versus the duration ratio between the contact and the period in the presence of CHR at the interface between the surfaces of rods with R_z = 3.2 μm from different materials and at different temperatures (P = 0.2 MPa; ν = 1.0 Hz): 1) steel 12Kh18N10T; 2) M2; 3) alloy D16T; 1, 2) T_c = 405 K; 3) 393; 4) 483.

Fig. 6. Heat resistance due to the periodic interruption of the thermal flow versus the duration ratio between the contact and the period in the presence of CHR at the interface between the surfaces of rods from different materials at different landed forces: 1, 2) steel 12Kh18N10T, R_z = 58 μm; 3–5) alloy D16T, R_z = 5.8 μm; T_c = 415 K, ν = 0.1 Hz; 1) P = 0.2 MPa; 2) 1; 3) 0.2; 4) 1.2; 5) 2.8.

$$R_c = \frac{S\Delta t}{C \ln \dfrac{\Delta T_1}{\Delta T_2}} . \qquad (46)$$

Then the obtained CHRs were transformed to the length of the portion of the rod l_c.

At the initial stage of realization of the experimental program, we investigated the process of formation of heat resistances for periodically contacting metallic surfaces at a minimum CHR. Such a process of heat exchange is characteristic of contact pairs with high-purity surfaces at a large landed force and small thermal flows, and in the presence in the contact zone of heat-conducting coatings or spacers [1].

Figure 3 presents the experimental data in dimensionless form. As objects of investigations, we used rods from stainless steel 12Kh18N10T with contact surfaces polished to a purity with $R_z \approx 0.8$ μm. A cadmium coating of thickness δ = 60 μm was applied by electroplating on the surface of the "hot" rod. The landed force of the rods during their contact was held at 0.08–0.16 MPa and the temperature — within the limits of 378 K. The contact heat resistance was thus minimized and equaled $R_c = 0.3 \cdot 10^{-4}$ m^2·K/W. From Fig. 3 it is seen that when $R_c \to 0$, the relation between the total heat resistance across the thermal flow in the form of a complex $\nu l_{int}^2/a^2$ and the contact frequency and duration described by the complex νt_1 is expressed by a single curve and confirms the correctness of relation (42). From this it also follows that with increasing frequency and at fairly large values of the complex $\nu\tau_c$ the losses of the thermal flow caused by its periodic interruption will be minimal.

Under real conditions it is necessary to take into account the effect of the final thermal CHR at the interface between the contacting rods. Apparently, due to the dependence of the CHR on a large number of factors the problem of experimental investigation of the process of heat exchange becomes much more complicated.

As follows from the works on the heat exchange [1–3], the strongest influence on the CHR is produced by the geometry of the contact surfaces. Contact pairs in which one surface (upper rod) was subjected to mechanical working by planing, milling, and polishing have been investigated. Experiments were performed at fixed frequency and landed force. The contact time of the rods was varied over a wide range. The results of the experiments are presented in Fig. 4. As would be expected, with increasing purity of contact surfaces the dimensionless complex characterizing the influence of the CHR decreases. It is also seen that the CHR-dependence of the total heat resistance to the thermal flow is the stronger the larger the duration ratio between the contact and the period νt_1.

Special experiments were performed for contact pairs from different metals and alloys. As is seen from the position of the curves in Fig. 5, the nature of the material of contact pairs has a certain influence on the formation of the total heat resistance to the thermal flow through periodically contacting surfaces. For instance, the pair from copper M2 has the least heat resistance. This is explained by its high heat conductivity and plasticity. The same figure also shows the influence of temperature conditions of the contact of surfaces from D16T alloy on the heat resistance. It is seen that an increase in the temperature in the contact zone from 393 to 483 K leads to a decrease in the CHR and total heat resistance. This is explained by the decrease in the hardness of the D16T alloy with increasing temperature. We also investigated the influence of such an important factor as the force landing on the surfaces of contact pairs. It is known that with increasing pressure the CHR decreases [1–3]. One would also expect in this case a decrease in the total heat resistance. Figure 6 presents the results of investigations with landed forces varied over the 0.2–2.8 MPa range. Contact pairs from steel 12Kh18N10T and alloy D16T were investigated. It is seen that an increase in the landed forces leads to a decrease in the CHR and, accordingly, in the total heat resistance. This effect is explained by the increase in the actual contact area of the surfaces and the decrease in the interfacial space with increasing landed forces.

Conclusions. The obtained results of theoretical and experimental investigations of the contact heat conduction through periodically contacting metallic surfaces makes it possible to gain an insight into the physical essence of the process of heat transfer and directionally regulate the formation of heat resistances to the thermal flow through such joints by changing the material of contact pairs, the geometry of contact surfaces, the temperature conditions, the landed forces, and the contact frequency and duration.

NOTATION

a^2, thermal diffusivity, m^2/sec; A_{in}, Fourier coefficient for $\tilde{u}_i$ at $t = 0$; B_{im}, Fourier coefficient for $\tilde{\mu}_i$ at $t = 0$; C, heat capacity of the rod material, J/K; $f^*(x)$, some auxiliary function $f_i(x)$, initial temperature in the right rod during the ith contact; H, dimensionless quantity; l, length of the rod, m; l_c, length of the portion of the rod; N, M, number of held terms in the Fourier series; P, landed force, MPa; Q, thermal flow, W/m^2; $Q_{\text{q.st}}$, stationary and quasi-stationary thermal flows; R_c, heat resistance, m^2·K/W; R_z, average height of projections of microinhomogeneities, μm; S, area, m^2; T, temperature, K; $\overline{T}$, relative temperature; T_0, initial temperature; $T_{i\text{int}}$, temperature in the ith cycle of the contactless state; $\tilde{u}_i$, auxiliary function for solving the problem on the temperature in the right rod during the ith contact; u_i, dimensionless temperature in the right rod during the ith contact after the ith separation; x_1, ..., x_5, coordinates of thermocouples; $\tilde{x}$, x, dimensional and dimensionless coordinates along the rod; δ, thickness, μm; $\varphi_i(x)$, initial temperature in the right rod in the ith separated state; λ, heat conductivity coefficient, W/(m·K); μ_i, temperature in the rod after the ith contact in the ith separated state; $\tilde{\mu}_i(x, t)$, auxiliary function for solving the problem on the temperature in the right rod during the ith separated state; ν, frequency, Hz; τ, time, sec; Δτ, time of the change in the temperature difference between the right and the left rod at the point of their contact by the value of ΔT_1 and ΔT_2, sec. Subscripts: c, contact; q.st, quasi-stationary; m, material of the rod; int, interruption; st, stationary; i, cycle number; m, n, numbers of terms in the Fourier series; z, average height of rough projections.

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